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Modified Homotopy Perturbation Method for Solving Benjamin- Bona- Mahony-Burger Equation

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Abstract

The Benjamin–Bona–Mahony–Burgers (BBMB) equation is an important nonlinear partial differential equation in fluid mechanics, used to describe the propagation of long waves. Due to its nonlinearity, obtaining exact solutions is difficult, which motivates the use of efficient approximate methods.In this paper, the BBMB equation is solved using the Variational–Homotopy Perturbation Method (VHPM), which combines the Variational Iteration Method (VIM) and the Homotopy Perturbation Method (HPM), providing a rapidly convergent series solution without requiring a small perturbation parameter. The results obtained by VHPM are compared with those of VIM and HPM. The solution is expressed as a series and truncated at the second-order term due to the method's fast convergence. Numerical examples are presented for both homogeneous and non-homogeneous cases, and the accuracy is evaluated using the error norms L_2 and L_∞.The results show that VHPM produces more accurate and stable solutions than the other methods. The reported error norms and graphical comparisons confirm the effectiveness and reliability of the proposed technique. Consequently, the VHPM is shown to be a powerful and efficient tool for solving the BBMB equation and similar nonlinear partial differential equations.

Keywords: Benjamin, Bona, Mahony, Berger's Equation, Variational, Homotopy Perturbation Method.

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طريقة الاضطراب الهوموتوبي المعدلة لحل معادلة بنجامن- بونا – ماهوني- برجر

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الملخص

تعد معادلة بنجامين–بونا–ماهوني–برغر (BBMB) من المعادلات التفاضلية الجزئية غير الخطية المهمة في ميكانيكا الموائع، إذ تستخدم على نطاق واسع لوصف انتشار الموجات الطويلة. وبسبب طبيعتها غير الخطية، فإن الحصول على حلول دقيقة لها يُعد أمرًا صعبًا، مما يحفّز استخدام طرائق تقريبية فعّالة.

في هذا البحث، تم حل معادلة BBMB باستخدام طريقة الاضطراب الهوموتوبي التغايري (VHPM)، التي تجمع بين طريقة التكرار التغاير (VIM) وطريقة الاضطراب الهوموتوبي (HPM) . توفر هذه الطريقة متسلسلة حلول سريعة التقارب دون الحاجة إلى معامل اضطراب صغير. وتمت مقارنة نتائج VHPM مع نتائج طريقتي VIM وHPM . يعبر عن الحل على هيئة متسلسلة ويُقتطع عند الرتبة الثانية نظرًا للتقارب السريع للطريقة. كما قُدّمت أمثلة عددية للحالتين المتجانسة وغير المتجانسة، وتم تقييم الدقة باستخدام معياري الخطأ L_2 و L_∞. تُظهر النتائج أن طريقة VHPM تعطي حلولًا أكثر دقة واستقرارًا مقارنة بالطرائق الأخرى. وتؤكد قيم معايير الخطأ والمقارنات البيانية فعالية وموثوقية التقنية المقترحة. وبناءً على ذلك، تُعد VHPM أداة قوية وفعّالة لحل معادلة BBMB وغيرها من المعادلات التفاضلية الجزئية غير الخطية المشابهة.

الكلمات المفتاحية: معادلة بنجامن- بونا- ماهوني- برجر، طريقة التغاير- الاضطراب الهوموتوبي.

Introduction

The Benjamin-Bona-Mahony-Burgers equation is considered one of the fundamental equations in fluid mechanics, a partial differential equation used in various fields of physics and applied

http://www.doi.org/10.62341/sbnh7914

mathematics, as it plays a vital role in describing the spread of long waves and depicting the behavior of nonlinear waves in fluids. This equation was originally introduced by (Peregrine, 1966), and (Benjamin et al, 1972). Discussed and provided a broad explanation of it. This equation combines the properties of the Burger equation, which represents dissipation, and the Benjamin-Bona-Mahony equation, which describes dispersion. The Benjamin-Bona-Mahoney-Berger (BBMB) equation takes this form

$$u_t - u_{xxt} - \alpha u_{xx} + \beta u_x + u u_x = g(x,t), \quad (x,t) \in \delta \times (0,T] \qquad (1)$$

With the boundary conditions

$$u(a,t) = u(b,t) = 0, \qquad t \in (0,T] \qquad (2)$$

The initial condition

$$u(x,0) = g_0(x), \qquad x \in \delta \qquad (3)$$

Where $\delta = [a,b]$ and $\alpha, \beta > 0$ are constant, u represents the wave height or amplitude at position x and time t, u_t is a partial time derivative (rate of change over time), u_x is the partial spatial derivative (horizontal gradient), u_{xxt} is the dispersion term (prevents singularities like "wave breaking" in solutions) and $g(x,t)$ is represents the non-homogeneous function (external source or disturbance). For $\alpha = 0$ the eq. (1) becomes a regularized long-wave (RLW/ BBM) equation.

Due to the complexity of the equation, many methods have been developed over time to solve it, combining analytical and numerical techniques. For example, (Jain, P.C. et al., 1993) applied splitting techniques using cubic B-splines. This was followed by the creation of the B-spline finite element method (Gardner, L.T.R. et al., 1995) and the least-squares finite element scheme (Gardner, L.T.R. et al., 1996). Meanwhile, (Soliman, A.A. et al., 2001) employed quadratic B-spline collocation methods. (Dogan, A., 2002) achieved a solution using the Galerkin method within finite elements in linear space. (Soliman A.A. et al., 2005) also used a combination of septic spline collocation. The finite difference method was explored by (Omrani, K., et al., 2007). Collocation methods continued to evolve (Zarebnia, M., 2013, 2016, 2017, 2020), with the implementation of cubic, quadratic, and quintic B-splines. Quartic B-spline collocation

http://www.doi.org/10.62341/sbnh7914

was developed by (Arora, Mittal, & Singh, 2014). Similarly, (Yagmurlu N.M. et al., 2018) solved the equation using the Strang splitting technique and finite element methods with cubic B-splines. (S.B.G. Karakoc S.B.G. et al., 2019) applied the Galerkin method to ensure the existence and uniqueness of solutions.

The primary objective of this paper is to obtain an approximate solution to this equation using the VHM method and the modified VHM method, which are effective and accurate in solving partial differential equations that are difficult to solve exactly.

2- The basics of methods

In this part, we will clarify the basic idea of the iterative methods that will be studied in this paper (Ghorbani A., 2010), (Al-Sawoor A.J. et al, 2013), (HE J.H, 1999), (Turq S., 2020).

2.1- The Variational Iteration Method

Consider the general nonlinear differential equation

$$L[u(x,t)] + N[u(x,t)] = g(x,t) \tag{4}$$

Where $L, N, g,$ respectively, are a linear differential operator, a nonlinear differential operator, and a given analytical function. According to VIM, we can construct a correction functional as

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(x,\tau)[Lu_n(x,\tau) + N\tilde{u}_n(x,\tau) - g(x,\tau)]d\tau \tag{5}$$

Where λ is a general Lagrange multiplier, which can be identified optimally via the variational theory, $\tilde{u}_n$ is a restricted variation, which means $\delta\tilde{u}(x,t) = 0$, where δ is the first variation. The approximate solution $u_{n+1}, n \geq 0$, of the solution $u(x,t)$can be established by determining a general Lagrange multiplier λ and using the initial approximation.

2.2- The Homotopy Perturbation Method

The basic idea of this method is a combination of the homotopy and perturbation methods. Hence, the method is referred to as the homotopy perturbation method (HPM).

Let us consider the following nonlinear differential equation

http://www.doi.org/10.62341/sbnh7914

$$A(u) - f(r) = 0, r \in \delta \quad (6)$$

With the boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \Gamma \quad (7)$$

Where A is a general differential operator, B is a boundary operator, r is a coordinate, $f(r)$ is a known function, and Γ is the boundary of the domain δ. Operator A can be divided into two operators: L (linear operator) and N (nonlinear operator).

Using the homotopy technique, we construct a homotopy $v(r,p): \delta \times [0,1] \rightarrow \Re$. Which satisfies

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \quad p\epsilon[0,1], \ r\epsilon\delta \quad (8)$$

Where u_0 is an initial approximation of Eq. (6), which satisfies the boundary conditions. Using the embedding parameter p as a small parameter, and assuming that the solution of equation (8) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots\cdots\cdots \quad (9)$$

Setting $p = 1$ gives the solution of equation (5)

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots\cdots \quad (10)$$

2.3- Variational- Homotopy Perturbation Method

Here, we will modify the variational Iteration method(VIM) by applying the Homotopy perturbation method (HPM) (9) to the functional correction equation (5), so we obtain

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = u_0(x,t) + p\int_0^t \lambda(x,\tau)\left[\sum_{n=0}^{\infty} p^n Lu_n(x,\tau) + \sum_{n=0}^{\infty} p^n N\tilde{u}_n(x,\tau)\right]d\tau - \int_0^t g(x,\tau)\, d\tau \quad (11)$$

Which is the Variational Homotopy Perturbation method. Comparing similar powers of P gives solutions of different orders, and we get the solution.

$$u = \lim_{p \to 1} \sum_{n=0}^{\infty} p^n u_n$$

http://www.doi.org/10.62341/sbnh7914

3- Application of iterations in the BBM-Burger equation

This section applies the iterations (HPM, VIM, VHM) in the BBM-Burger equation, where

$$L[u(x,t)] = u_t$$

$$N[u(x,t)] = -(u_{xxt} + \alpha\, u_{xx} - \beta u_x - u\, u_x)$$

Or

$$A(u) = u_t - u_{xxt} - \alpha u_{xx} + \beta u_x + u u_x$$

3.1- The Application of VIM in the BBM-Burger equation

In this part, we will present an approximation solution of the BBM-Burger equation. (1) using VIM.

First, we will write the correction functional (5) as follows

$$u_{n+1} = u_n + \int_0^t \lambda(x,\tau)(u_{n\tau} - \tilde{u}_{nxx\tau} - \alpha\, \tilde{u}_{nxx} + \beta\, \tilde{u}_{nx} + \tilde{u}_n\, \tilde{u}_{nx} - g(x,\tau))d\tau \qquad (12)$$

Where $\tilde{u}_n$ is a restricted variation, $n \geq 0$.

Making the correction functional (1) stationary, we notice that $\delta\tilde{u}_n = 0$. Therefore, we have

$$\delta u_{n+1} = \delta u_n + \delta \int_0^t \lambda(u_{n\tau})d\tau$$

So, the stationary condition can be obtained as follows

$$1 + \lambda(\tau) \downarrow_{\tau=t} = 0$$

$$\frac{\partial\lambda}{\partial\tau} \downarrow_{\tau=t} = 0$$

The Lagrange multipliers can be identified as $\lambda = -1$. So, the iteration formula is given as:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t (u_{n\tau} - u_{nx\tau\tau} - \alpha\, u_{nxx} + \beta u_{nx} + u_n u_{nx} - g(x,\tau))d\tau \qquad (13)$$

Therefore, we have

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

http://www.doi.org/10.62341/sbnh7914

3.2- Homotopy- BBM-Burger equation:

Here, we apply the HPM to solve the BBM-Burger eq. (1). Then, by using the homotopy technique, we have

$$H(u,p) = (1-p)(u_t - u_{0t}) + p(u_t - u_{xxt} - \alpha u_{xx} + u_x + uu_x - g(x,t)) = 0 \quad (14)$$

Or

$$H(u,p) = u_t - u_{0t} + p(u_{0t} - u_{xxt} - \alpha u_{xx} + \beta u_x + uu_x - g(x,t)) = 0 \quad (15)$$

Substituting the initial condition and $u = \sum_{n=0}^{\infty} p^n u_n$

We get

$$(p^0u_0 + p^1u_1 + \cdots)_t - u_{0t} + p[u_{0t} - (p^0u_0 + p^1u_1 + \cdots)_{xxt} - \alpha(p^0u_0 + p^1u_1 + \cdots)_{xx} + \beta(p^0u_0 + p^1u_1 + \cdots)_x + (p^0u_0 + p^1u_1 + \cdots)(p^0u_0 + p^1u_1 + \cdots)_x - g(x,t)] = 0 \quad (16)$$

By comparing the coefficients of terms with identical power p, we get

$$p^0: u_{0t} - u_{0t} = 0$$
$$p^1: u_{1t} = -u_{0t} + u_{0xxt} + \alpha u_{0xx} - \beta u_{0x} - u_0u_{0x} + g(x,t))$$
$$p^2: u_{2t} = u_{1xxt} + \alpha u_{1xx} - \beta u_{1x} - u_0u_{1x} - u_1u_{0x} \quad (17)$$
$$p^3: u_{3t} = u_{2xxt} + \alpha u_{2xx} - \beta u_{2x} - u_0u_{2x} - u_1u_{1x} - u_2u_{0x}$$

Similarly, we can derive the remaining terms.

Therefore, we have

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$

3.3- VHM- BBM-Burger equation:

We will apply the V.H.M. to solve the VHM-BBMB equation. The correction functional is given as

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t \lambda(x,\tau)[u_{n\tau} - \tilde{u}_{nxx\tau} - \alpha\,\tilde{u}_{nxx} + \beta\tilde{u}_{nx} + \tilde{u}_n\tilde{u}_{nx} - g(x,\tau)]d\tau \quad (18)$$

Where $\tilde{u}_n$ is a restricted variation.

http://www.doi.org/10.62341/sbnh7914

Making the correction functional (15), the Lagrange multiplier can be determined as $\lambda = -1$

Thus, the iteration formula is given as

$$u_{n+1}(x,t) = u_0(x,t) - \int_0^t [u_{n\tau} - u_{nxx\tau} - \alpha\, u_{nxx} + \beta u_{nx} + u_n u_{nx} - g(x,\tau)]d\tau \quad (19)$$

Applying the V.H.M, we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = u_0(x,t) - p\int_0^t [(\sum p^n u_n)_\tau - (\sum p^n u_n)_{xxt} - \alpha(\sum p^n u_n)_{xx} + \beta\ (\sum p^n u_n)_x + (\sum p^n u_n)\ (\sum p^n u_n)_x - g(x,\tau)\ ]d\tau \quad (20)$$

By comparing the coefficients of similar powers of P, we get the solutions of different orders

$$p^0: u_0 = u_0(x,0)$$

$$p^1: u_1(x,t) = -\int_0^t [u_{0\tau} - u_{0xx\tau} - \alpha u_{0xx} + \beta u_{0x} + u_0 u_{0x} - g(x,\tau)]d\tau$$

$$p^2: u_2(x,t) = -\int_0^t [u_{1\tau} - u_{1xx\tau} - \alpha u_{1xx} + \beta u_{1x} + u_1 u_{0x} - u_0 u_{1x}]d\tau \quad (21)$$

$$p^3: u_3(x,t) = -\int_0^t [u_{2\tau} - u_{2xx\tau} - \alpha u_{2xx} + \beta u_{2x} + u_0 u_{2x} - u_1 u_{1x} + u_2 u_{0x}]d\tau$$

So ,we have

$$u_0(x,t) = u_0(x,0)$$

For $n \geq 1$

$$u_n(x,t) = -\int_0^t \left[u_{\mathrm{n-1}\tau} - u_{\mathrm{n-1}xx\tau} - \alpha u_{\mathrm{n-1}xx} + \beta u_{\mathrm{n-1}x} + \sum_{k=1}^{n-1} u_k u_{n-1-kx} - \delta_{n,1} g(x,\tau) \right] d\tau$$

Where $\delta_{n,1}$kronecker delta function, and

http://www.doi.org/10.62341/sbnh7914

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$

4- Numerical Example

In this section, we explore various examples to demonstrate the effectiveness of the VIM, HPV and VHPM methods in solving the BBMB (eq.1) with initial condition (3) for $\alpha = \beta = 1$. These examples are chosen from (G. Arora G. et al., 2014), (Shallu et al., 2022), (Izadi, M. et al., 2022). To prove the effectiveness and efficiency of the methods presented in this manuscript to solve equation 1 in terms of accuracy, stability and arithmetic complexity through tables and graphs and calculating error norms L_2 and L_∞ for different times t via

$$L_2 = \|u - u_{exact}\|_2 = \sqrt{\Delta t \sum_{i=0}^{N} \left(u(x,t_i) - u_{exact}(x,t_i)\right)^2}$$

$$L_\infty = \|u(x,t_i) - u_{exact}(x,t_i)\|_\infty = \max_{0 \le N \le i} |u(x,t_i) - u_{exact}(x,t_i)|$$

All numerical computations presented in this work are performed using the second-order approximation. This choice is justified by the rapid convergence of the Methods in this paper, where higher-order terms contribute marginally to the solution accuracy while increasing the computational effort

This is achieved by following the steps outlined in Section 3 and Equations 13, 17 and 21 to obtain the required results.

Example [1]

We consider the homogeneous BBMB (eq.1) subject to the initial condition

$$u(x,0) = e^{-x^2}$$

http://www.doi.org/10.62341/sbnh7914

Since the exact solution to this equation is unknown, we will compare the approximate solution u_2 of this equation in different intervals. From Figures 1, 2, and 3, we notice that the solutions are almost identical. However, when the space intervals are larger, we find a slight difference in the solutions, where the methods VIM and HPM yield the same results for u_2. As for the VHPM, additional terms arise from the integration of variational principles with the HPM methodology.

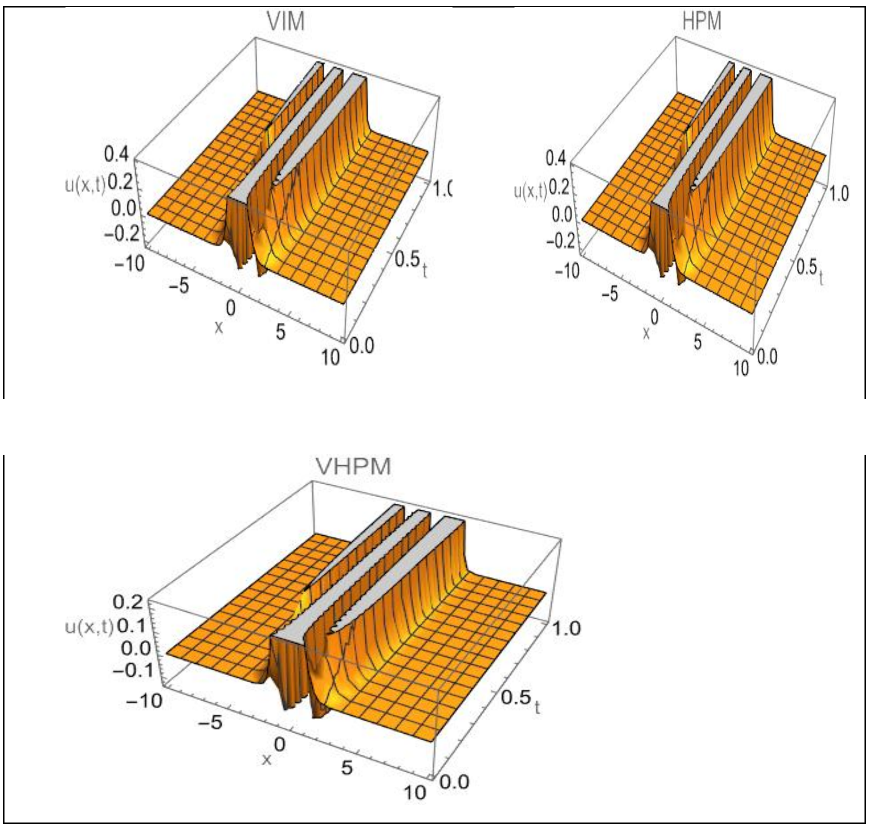

Fig1: 2-attractive approximation at $(x,t) \in [-10,10] \times [0,1]$

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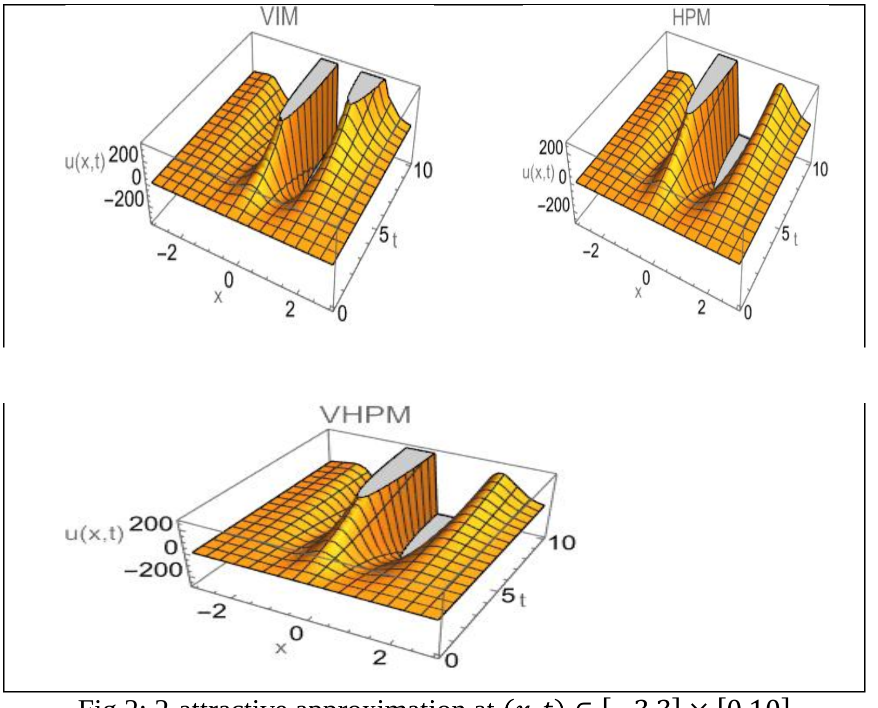

Fig 2: 2-attractive approximation at $(x,t) \in [-3,3] \times [0,10]$

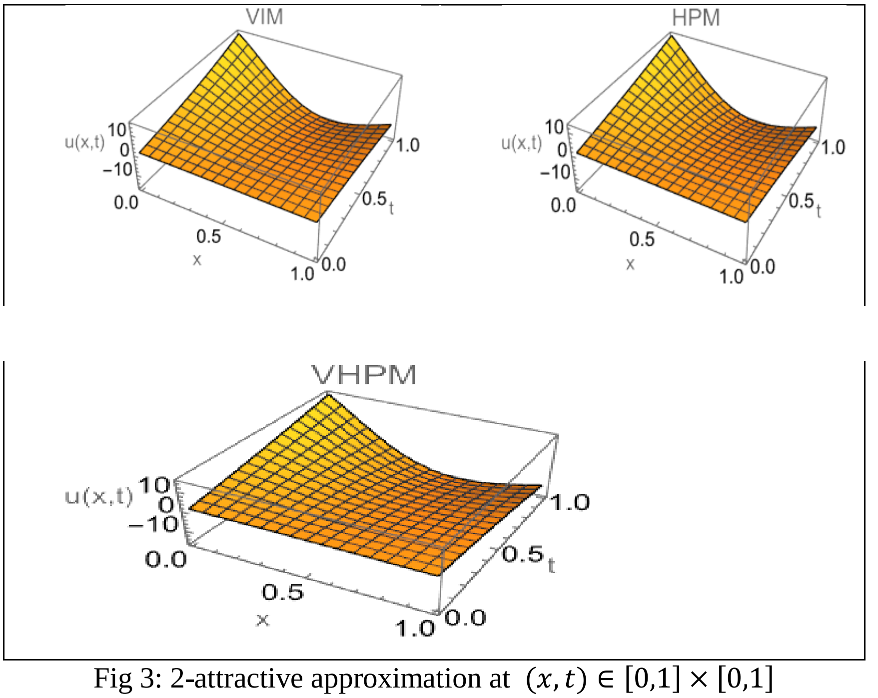

Fig 3: 2-attractive approximation at $(x,t) \in [0,1] \times [0,1]$

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Example [2]

We report the non-homogeneous BBMB (eq.1) with the initial condition:

$$u(x,0) = \sin x$$

In which the range $x \in [0,\pi]$, $g(x,t) = e^{-t}\left(\cos x - \sin x + \frac{1}{2}e^{-t}\sin 2x\right)$.

The exact solution is $u(x,t) = e^{-t}\sin x$

Table 1 and Fig. 4 compare the exact and approximate solutions, using L_2 and L_∞ norms for different values of t and $\Delta t = 0.01\ with\ N = 121$.

Table 1: The error norm for different times t

T	VIM		HPM		VHPM	
	L_2	L_∞	L_2	L_∞	L_2	L_∞
1	6.374×10^{-6}	6.158×10^{-6}	9.241×10^{-6}	8.927×10^{-6}	5.812×10^{-6}	5.623×10^{-6}
2	3.821×10^{-6}	3.729×10^{-6}	$5{,}673 \times 10^{-6}$	5.482×10^{-6}	3.415×10^{-6}	3.342×10^{-6}
4	8.158×10^{-7}	7.124×10^{-7}	1.283×10^{-6}	1.142×10^{-6}	6.927×10^{-7}	6.158×10^{-7}
10	3.827×10^{-8}	3.513×10^{-8}	6.452×10^{-8}	5.927×10^{-8}	3.142×10^{-8}	2.885×10^{-8}

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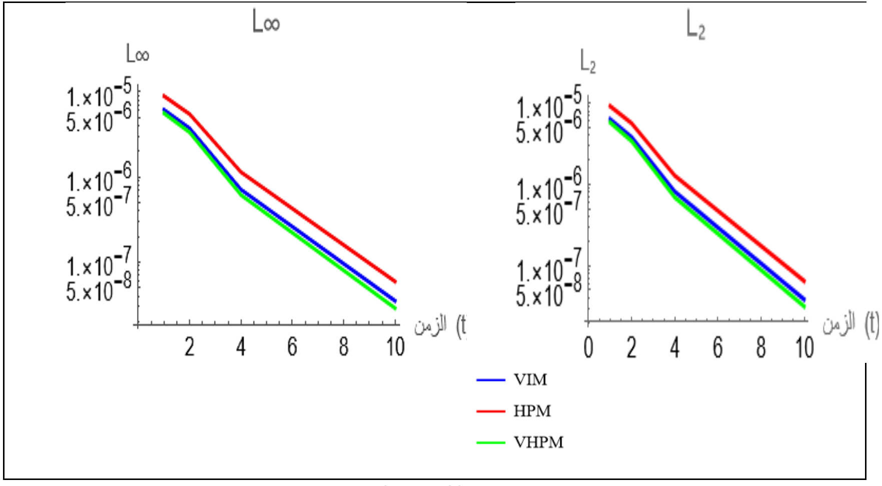

Fig 4: the error norm for different times t and N=121

Conclusions

In this paper, we present the Modified Homotopy Perturbation Method, which couples the HPM with the VIM, to solve the BBMB equation. Through the provided examples and mathematical analyses, the results showed that the VHPM offers an effective approach, serving as a powerful tool for accurate analytical solutions to this equation, as it outperformed the accuracy of the VIM and the speed of the HPM. Through Figures 1, 2, and 3, we observe that the time value t influences the displayed methods. In addition to that, Table 1 reports the values of the L_2 and L_∞ error norms at different time levels. It is observed that the VHPM yields smaller error norms compared to the VIM and HPM, indicating better accuracy and stability. Moreover, the error remains bounded as time increases, confirming the reliability of the obtained approximation. From the obtained figures and table 1, we find that the second approximation (u_2) was sufficient for accurate results, as higher-order terms showed negligible contribution, ensuring computational efficiency. Higher-order approximations ($u_3, u_4, \cdots$) did not significantly improve accuracy but increased computational cost. The rapid convergence of the methods and the small errors obtained with (u_2) confirm its adequacy for this problem.

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